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Note on Mr. Stone's Paper, "Expressions for the Elliptic Coordinates of a Moving Point to the Seventh Order of Small Quantities." By Professor Ernest W. Brown.

In the January number of the *Monthly Notices* Mr. Stone gives the expressions for the polar coordinates of a point moving in an ellipse about one focus, as far as quantities of the seventh order inclusive, with respect to the eccentricity and inclination. It may perhaps be useful to mention that these expressions are substantially contained in a memoir by Cayley, "Tables of the Developments of Functions in the Theory of Elliptic Motion" (*Memoirs R.A.S.* vol. xxix. 1861, pp. 191–306; *Coll. Works*, vol. iii. pp. 360–474). It is true that the expressions there given refer only to the case of motion in the plane of reference, but the steps necessary to obtain the longitude and latitude when the ellipse is inclined to the plane of reference require little more than the reading off of the coefficients from the tables.

The expressions for positive and negative powers of r/a are given on pp. 425–427 (*Coll. Works*), and that for the orbital elliptic longitude on p. 474.

To obtain the longitude in the plane of reference, we have, in Delaunay's notation,

$$\tan(\mathrm{V}-h)=\cos i\tan\nu;$$

whence

$$\mathrm{V}-h=\nu-\tan^2\frac{1}{2}i\sin 2\nu+\frac{1}{2}\tan^4\frac{1}{2}i\sin 4\nu-\frac{1}{3}\tan^6\frac{1}{2}i\sin 6\nu+\ .\ .$$

or

$$\mathrm{V}=h+\nu-(\gamma^2+\gamma^4+\gamma^6)\sin 2\nu+\left(\frac{1}{2}\gamma^4+\gamma^6\right)\sin 4\nu-\frac{1}{3}\gamma^6\sin 6\nu+\ .\ .\ .\ (1)$$

Put

$$\nu=g+f,$$

where f is the true anomaly, and let

$$\cos jf=\Sigma_i\mathrm{A}_i\cos il,\qquad \sin jf=\Sigma_i\mathrm{B}_i\sin il,$$

where i, j are positive integers. We obtain

$$\sin j\nu=\frac{1}{2}\Sigma_i(\mathrm{A}_i+\mathrm{B}_i)\sin(jg+il)+\frac{1}{2}\Sigma_i(\mathrm{A}_i-\mathrm{B}_i)\sin(jg-il).$$

The coefficients

$$\frac{1}{2}(\mathrm{A}_i+\mathrm{B}_i),\qquad \frac{1}{2}(\mathrm{A}_i-\mathrm{B}_i)$$

are tabulated by Cayley * for all values of i, j from 1 to 7, as far as the order e^7 inclusive; so that the coefficient of any term in

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* His notation differs from that used above.

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V is obtained by taking out the part dependent on the eccentricity from the tables and multiplying it by the corresponding coefficient in (1).

To obtain the latitude, we have

$$\sin \mathrm{U} = \sin i \sin \nu ;$$

whence (Hobson's *Trigonometry*, p. 268)

$$\mathrm{U} = \sin i \sin \nu + \frac{1}{6} \sin^3 i \sin^3 \nu + \frac{3}{40} \sin^5 i \sin^5 \nu + \frac{5}{112} \sin^7 i \sin^7 \nu + \ . \ .$$

Putting

$$\gamma = \sin \frac{1}{2} i,$$

this gives

$$\mathrm{U} = \left(2\gamma - \frac{1}{4}\gamma^5 - \frac{3}{8}\gamma^7\right) \sin \nu - \left(\frac{1}{3}\gamma^3 + \frac{1}{4}\gamma^5 + \frac{1}{8}\gamma^7\right) \sin 3\nu$$
$$+ \left(\frac{3}{20}\gamma^5 + \frac{1}{4}\gamma^7\right) \sin 5\nu - \frac{5}{56}\gamma^7 \sin 7\nu + \ . \ . \ .$$

which is expanded, by means of Cayley's tables, in terms of sines of multiples of g, h and powers of e, γ, in the same way as the corresponding terms of the longitude.

It will be noticed that the coefficients of γ^7 in $\sin \nu$ and $\sin 3\nu$ differ from those obtained by Mr. Stone. He finds

$$+\frac{3}{8}\gamma^7 \sin (g + l), \qquad -\frac{3}{8}\gamma^7 \sin 3(g + l).$$

I obtain, by the above method, for these terms

$$-\frac{3}{8}\gamma^7 \sin (g + l), \qquad -\frac{1}{8}\gamma^7 \sin 3(g + l).$$

respectively. I have verified all the expressions and, with the exception of the terms just mentioned, find that the results agree. The fact that this verification occupied about half an hour will show the great convenience of Cayley's tables.

It may be mentioned that an evident typographical error occurs on p. 377 of the tables (*Coll. Works*, vol. iii.), where the coefficient $\frac{765}{1024}$, in the first line of the second column, is printed $\frac{768}{1024}$. This error is corrected by means of the three coefficients immediately below in the same column.

Haverford College, Pa.:
1895 *February* 24.

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Note on Professor Brown's Note. By E. J. Stone.

Professor Brown is quite right in saying that the terms

$$\sin(g+l)\left[+\tfrac{3}{8}\gamma^7\right] \text{ and } \sin(3g+3l)\left[-\tfrac{3}{8}\gamma^7\right]$$

which appear on p. 113 in my paper, "Expressions for the Elliptic Coordinates of a Moving Point to the Seventh Order of Small Quantities," are incorrectly printed.

The first is simply an error of sign. The second has arisen from the coefficient found for sin $(g+l)$, $-\frac{3}{8}\gamma^7$, having been by inadvertence copied by me, when preparing the copy for press, into the argument for sin $(3g+3l)$, instead of $-\frac{1}{8}\gamma^7$.

The coefficients of the terms sin $(5g+5l)$ and sin $(7g+7l)$, which involve γ^7, and were found conjointly with those above-mentioned, are correctly printed.

Radcliffe Observatory, Oxford:
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On the Relative Efficiency of a Reflector and of Portrait Lenses for the Delineation of Celestial Objects. By Isaac Roberts, D.Sc., F.R.S.

It has often been asserted that portrait lenses have, by reason of their short foci and relative apertures, greater photographic power than instruments constructed on other models, such as a reflector; but the assertion had not been confirmed, on any occasion, within my experience in the past. I therefore considered it desirable to submit the question to the test of some crucial experiments; the results of which shall now be laid before the Society.

The experiments were conducted in the manner following.

Two typical portrait lenses were purchased, of the best quality that could be obtained, and were affixed with their cameras firmly to the tube of the 20-inch reflector. The arrangements were such that three photographs of the same objects, by three different instruments, could be taken simultaneously under precisely and in every respect similar conditions.

The photo-plates were of the same degree of sensitiveness, the durations of the exposures were quite equal, and the development of the negatives was similar in all cases.